Allocation of Dynamic Operating Envelopes in Distribution Networks: Technical and Equitable Perspectives

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***Abstract*—A Dynamic Operating Envelope (DOE) specifies the available capacity to import/export power for Distributed Energy Resources (DERs) or connection points of a distribution network in a time interval without violating its physical and operational constraints. This paper proposes a two-stage, top-down approach to allocate DOEs in MV-LV (medium voltage-low voltage) integrated distribution networks. In the first stage, Distribution System State Estimation (DSSE) and Capacity Constrained State Optimisation (CCSO) techniques are employed to allocate DOEs at transformer connection points. Using network data from available metering infrastructure, DSSE estimates the current operational state of the network, whereas CCSO facilitates the DOE allocations at transformer connection points considering technical, soft-equitable and equitable perspectives. In the second stage, the allocated DOEs, containing available aggregate export/import power, are distributed among DERs of LV networks ensuring network integrity. Real-world data of an Australian MV-LV network is employed to demonstrate the effectiveness of proposed approach. Test results show that technical perspective can offer maximum allocated export/import power, whereas, minimum disparity of allocated power among different connection-points is attained for equitable perspective. Further, insightful performance studies of DOE and existing fixed export policies are conducted considering the condition of present as well as future power systems expecting substantial proliferations of renewables.**

***Index Terms*—dynamic operating envelope, distribution system state estimation, optimisation, network's constraints, distributed energy resources.**

NOMENCLATURE

Notation

$\Re(\cdot), \Im(\cdot)$	real and imaginary parts of a complex number
H	set of households (all), indexed by h
$H^*(\subset H)$	set of households representing DOE-registered prosumers, indexed by h^*
$\mathcal{N}$	set of 3-phase buses in the LV network
$\mathcal{N}_{h^-}(\subset \mathcal{N})$	set of 3-phase buses connected with passive customers
$\mathcal{N}_{h^+}(\subset \mathcal{N})$	set of 3-phase buses connected with fixed export (FE) prosumers
$\mathcal{N}_{h^*}(\subset \mathcal{N})$	set of 3-phase buses connected with DOE prosumers
Φ	set of phases $\{a, b, c\}$
$\Phi_k(\subseteq \Phi)$	set of bus-k phases
$\Phi_{k,h^-}(\subseteq \Phi)$	set of bus-k phases connected with passive customers
$\Phi_{k,h^+}(\subseteq \Phi)$	set of bus-k phases connected with fixed-export prosumers
$\Phi_{k,h^*}(\subseteq \Phi)$	set of bus-k phases connected with DOE prosumers
Δp_k^s	available import/export power to/from the house at phase $s \in \Phi_{k,h^*}$ of bus $k \in \mathcal{N}_{h^*}$
Δp_{total}	available import/export power at the LV transformers' end in a given time-interval
$\tilde{p}_k^s$	measured active power at the house connected at phase s of bus k
$\tilde{q}_k^s$	measured reactive power at the house connected at phase s of bus k
v_k^s	complex voltage at bus-k of phase s
$G_{kj}^{s\gamma}$ and $B_{kj}^{s\gamma}$	conductance and susceptance of nodal admittance matrix of LV network
$\underline{v}$ and $\bar{v}$	lower and upper limit of voltage (statutory)
$\Delta p_k^{s,opt}$	Optimal allocation of available export/import power at the house connected at phase $s \in \phi_{k,h^*}$ and bus $k \in \mathcal{N}_{h^*}$
$\Delta p_k^{s,\text{equi}}$	Equitable allocation of available export/import power at the house connected at phase $s \in \phi_{k,h^*}$ and bus $k \in \mathcal{N}_{h^*}$
ΔP_i^{exp}	Available export power at the distribution transformer i
ΔP_i^{imp}	Available import power at the distribution transformer i

I. INTRODUCTION

THE traditional electricity networks were designed for one-way aggregate energy flows where the physical and operational limits of the network are respected for a well characterised energy consumption pattern. However, over the last decade, electricity distribution networks have experienced dynamic two-way flows of energy due to high uptake of un-orchestrated distributed energy resources (DERs), such as, solar photovoltaic (PV), small/medium-scale batteries, etc. At times, reverse power flow, too low or too high net demand, may impose threat on the

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physical and operational limits. In this context, the impact of rooftop solar PV has been assessed in literature for the last decade [1-4]. Therefore, to mitigate network's statutory limit violations, static import/export limits on connection points have been brought into practice. For instance, Energex, the distribution network operator in South East Queensland, Australia, enforces 5 kW export limits and 10 kW import limits for customer connections [5]. However, these static limits are based on worst-case loading and generation scenarios that rarely occur in practice. Thus, these conservative limits may lead to underutilisation of consumer-owned DERs in low-voltage (LV) networks.

In addition to static limits, a potential solution through active network management strategies (including volt/var and volt/watt settings in the inverters) have been brought into practice as per the rules set out in AS/NZS 4777.2 and IEEE 1547.2 standards [6-7]. These strategies are focused on providing set-point control for DERs that will best allow some operational objectives to be achieved. Though these strategies can ensure network integrity for substantial period of time, overall network integrity may still not be guaranteed at all times.

To alleviate these issues, a novel technique, namely, Dynamic Operating Envelope (DOE), has been introduced that can efficiently utilise existing electricity infrastructure while appropriately managing distribution network's constraints. As per the outcome study report on DOEs, published by the Australian Renewable Energy Agency (ARENA), DOEs are defined as "operating envelopes [that] vary import and export limits over time and location based on the available capacity of the local network or power system as a whole" [8]. Overall, the benefits of DOEs can be summarised as: 1) DOE enable more solar and battery power exports to the grid, 2) it provides greater market efficiency under greater network capacity leading to a reduction of wholesale electricity market prices [9], and 3) it can minimise the need for costly supply-side upgrades under greater network asset utilisation and efficient use of the existing network. According to [10], DOEs can be implemented at a customer connection point or at the terminals of DER assets. For customer connection points, the total behind-the-meter power flows are constrained by the operating envelope. In case of DER assets, DOEs provide a range on DERs' active and reactive power set points (referred to as nodal limits on real and reactive power injection or demand) ensuring that, physical and operational limits of the network are not breached. Compared to the conventional approach where static import-export limits are determined at the customer connection point, DOEs accommodate greater utilisation of the existing electrical infrastructure whilst honouring network statutory limits. For graphical illustration, a schematic view of DOE is depicted in Fig. 1.

Considering the implementation of DOEs in low-voltage (LV) networks, the authors in [11] have proposed an approach for the wholesale market participation of DERs in an MV-LV integrated network. In this method, at each time step, the distribution network operator (DNO) performs a load flow to verify network constraint violations, and if exist, a three-phase snapshot Optimal Power Flow (OPF) problem is solved to assign dynamic export limits for households. Otherwise, the planned household operation remains unchanged. Furthermore, each household solves a typical home energy management problem considering the battery storage to be the controllable DER asset. In [12], a network-aware scheduling scheme for end-users in day-ahead markets is introduced. First, the DNO solves a three-phase OPF problem to calculate envelopes for end-users. In the next step, households send information on envelopes to the aggregator. Finally, the aggregator determines the schedule of battery storage based on envelopes calculated by the DNO. In [13], the authors have demonstrated a real-world implementation of a two-level control approach for local network management. At the DNO level, a multi-period OPF-scheduling problem is solved, and at the prosumer-level, household battery storage is controlled to limit the active power at the customer point-of-connection. A further extension of the work in [13] is proposed in [14] where network-secure envelopes are proposed for market participation of end-users in LV residential networks. In addition to active power control at the connection point, a Q-P controller is also proposed for the commitment of reactive power. Despite the accuracy and computational performance, the applicability of the approach is only limited to assigning DOEs for downstream nodes of a particular MV/LV transformer.

As opposed to solving an optimisation problem in OPF-based schemes, calculating network-sensitivity factors using analytical and regression-based methods and thereby determining power limits at end-user connection points is also exploited in the existing literature. For instance, the authors in [15] have presented a methodology to integrate DNOs with aggregators for secure scheduling and real-time operation of demand response (DR) in LV networks. In doing so, a regression-based method is employed to determine sensitivities of active and reactive power with respect to network states, e.g., voltages and currents. In [16], numerically calculated network sensitivity coefficients are used to determine active and reactive power set-points for DERs and thereby to develop a two-layer framework for the day-ahead scheduling and real-time control operation in wholesale markets. Effective distribution system management has been reported in [17-18] by incorporating the control decisions of battery energy storage (BES) systems considering the network constraints. However, the implementation of these methods poses a substantial challenge as the underlying assumptions of these studies could create regulatory issue between DNO and a third-party (e.g., aggregator) [11].

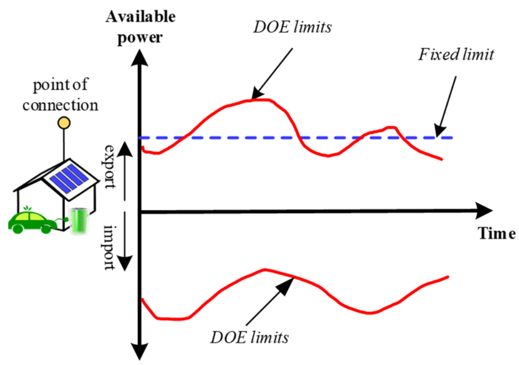

Fig. 1. A schematic diagram illustrating the available export-import power of DOE.

To estimate DOE for MV-LV integrated networks, most of the reported approaches in literature have assumed the availability of error-free meter-data at each time-interval considering full observability and then, OPF is conducted to determine dynamic operating envelopes. For example, in [11], an integrated MV-LV

OPF problem is solved to assign operating envelopes for DER connections. However, this approach often leads to a formulation of an OPF problem with a large number of decision variables representing nodal voltages, power set-points for end-users, and therefore, is computationally challenging. On the other hand, the meter-data from real MV-LV networks may be impacted by measurement deficiency (e.g., meter error, communication failure, etc.) at some/most of the time-intervals. For such plausible situations, an approach similar to MV-LV OPF as introduced in [11] — with the assumption of fully observable network estimated from error-free meter-data — may not be adopted to determine DOEs with precision. To alleviate these challenges and, to aim for a robust practical implementation, a novel two-stage, top-down approach is proposed in this article for allocating DOEs for households.

The first stage of the overall two-stage approach exploits Distribution System State Estimation (DSSE) [19] technique to acquire operational states of the MV network using real meter-data with a balanced representation of the physics of the network. Traditional distribution networks are typically equipped with minimum meters with limited visibility/observability. Strategically, state estimation can point out the unobservable locations and can ensure the full observability with the inclusion of branch measurement devices in those unobservable locations [19]. Hence, DSSE can effectively ensure the accuracy in voltage and angle estimation considering the theoretical as well as real-field aspects. Thus, the choice of DSSE is mainly due to the tolerance to partially under-determined network equipped with real measurements which could be impacted by measurement deficiency. A Capacity Constrained State Optimisation (CCSO) problem is then adopted to calculate DOEs at each MV/LV transformer of the network.

In the second stage, the allocated transformer-level DOEs are distributed optimally among DOE-enabled households in each LV network via an unbalanced AC OPF approach. Unlike [11] where a single OPF problem is solved to allocate DOEs among households, our two-stage approach is favourable in the sense that it readily allows parallel evaluation of OPF at LV networks at each stage with less decision variables and therefore, reduces the computational time. On the other hand, by not limiting the decision-making process to a single objective as in [11,13,14], our approach provides the flexibility to the DNO to adopt diverse business strategies: technical, soft-equitable, and equitable, to assign DOEs at distribution transformers' end. Furthermore, the formulation of the CCSO problem in the first stage fits well with fairly balanced MV networks and the unbalanced AC OPF formulation in the second stage accurately captures inherent unbalanced nature in typical LV networks.

In summary, the key contributions of the paper are:

1) The optimisation platform of proposed framework offers the flexibility to investigate technical, soft-equitable and equitable perspectives for DOE allocations. Thus, depending on the evolving network states and/or business strategies, the perspectives of DOE allocations could be adjusted/changed accordingly.

2) The available export/import power at LV networks have been allocated among DOE prosumers ensuring reasonable fairness that can be achieved under equitable DOE allocation.

3) The proposed approach offers parallel evaluation of unbalanced AC-OPF for the individual LV networks and thus, reduces the computational time.

The remaining part of the paper is organised as follows. Section II discusses the formulation of the problem to estimate DOEs. The proposed methodology is elaborated in Section III. A comprehensive case study using a real MV-LV integrated network and its historical data is detailed in Section IV. Further, the test results followed by insightful discussions are presented in Section IV, considering the current as well as future power systems enriched with high renewables. Discussions and final conclusions are detailed in Sections V and VI, respectively.

II. Problem Formulation

In the first stage of our proposed approach, a CCSO problem coupled with DSSE is formulated to assign DOEs at MV/LV transformers' end of MV network. The CCSO-based formulation further takes into account of diverse business strategies for the DNO in allocating DOEs at distribution transformers' end. In the second stage, an unbalanced AC-OPF problem is formulated at the LV network to assign set-points for DOE-enabled DERs, respecting the physical and operational limits of the LV network. Please note, DSSE is estimated by adopting the methodology presented in [19]; please refer to [19-23] for detailed discussions on state estimation and DSSE. This section formulates the CCSO problem with detailed explanations in section II-A.1) and section II-A.2); the problem in regards to allocating available export/import power for DOE-enabled prosumers is formulated in section II-B.

A. Formulation of the CCSO problem

Capacity constrained state optimisation (CCSO) is an alternative approach to determine optimum utilisation of a particular network by loads and generators, respecting the operational and technical limits of the network [19]. In other words, the CCSO problem determines a set of linear inequalities on decision variables which describes the control subspace that contains all feasible combinations of control variable set-points at a particular operating state of the network. Since MV networks are fairly balanced, the CCSO approach is utilised to determine DOEs at distribution transformer level of the MV network.

1) Capacity Constrained Operational Parameters

Consider the steady-state operation of an n-bus distribution network with the input vector $\boldsymbol{\alpha}$ given by

$$\boldsymbol{\alpha} \equiv (|V_1|, \delta_1, \boldsymbol{u}, \boldsymbol{w}) \quad (1)$$

$$\boldsymbol{u} \equiv \left(\boldsymbol{u}_1, \cdots, \boldsymbol{u}_k, \cdots, \boldsymbol{u}_{K_{nc}}\right)^T, \quad \boldsymbol{u}_k \equiv (P_k, Q_k)^T, \quad \forall k \in \mathcal{K}_{nc} \quad (2)$$

$$\boldsymbol{w} \equiv \left(\boldsymbol{w}_1, \cdots, \boldsymbol{w}_k, \cdots, \boldsymbol{w}_{K_c}\right)^T, \quad \boldsymbol{w}_k \equiv (P_k, Q_k)^T, \quad \forall k \in \mathcal{K}_c \quad (3)$$

where $\mathcal{K}_c$ is the set of controllable PQ buses (containing control variables), and $\mathcal{K}_{nc}$ is the set of uncontrollable PQ buses. Also, K_c and K_{nc} are the number of buses in the sets $\mathcal{K}_c$ and $\mathcal{K}_{nc}$, respectively. Please note, in (1)-(3), the buses connected with DERs are referred to as controllable PQ buses, whereas, the buses without DERs are referred to as uncontrollable PQ buses.

The complex voltage vector at each node, which represent the internal state of a network, can be estimated by state estimation approach taking $\boldsymbol{\alpha}$ as inputs [20-23]. Thus, the estimated state, $\mathbf{S}$, of the entire network can be given by

$$\mathbf{S} \equiv (|V|_1, \delta_1, \cdots, |V|_k, \delta_k, \cdots |V|_n, \delta_n), \quad \forall k \in \mathcal{N} \quad (4)$$

where $\mathcal{N} = \{1, \cdots, n\}$ is the set of all buses; and, alternatively, it can be presented as $\mathcal{N} = \{1\} \cup \mathcal{K}_c \cup \mathcal{K}_{nc}$.

The impact of control variables, $\mathbf{w}$, on the constrained output variables (e.g., voltage, line current, etc.), can be represented as output vector $\boldsymbol{\beta}$, which is comprised of non-analytic and non-linear functions. Therefore, an analytical Jacobian approach, as reported in [24], is adopted in this article to establish the linear map between control variables and constrained output variables. To this end, Jacobian matrix of the constrained output variables' functions can be computed from (5)

$$\mathbf{J}_{\boldsymbol{\beta}} = \begin{bmatrix} \frac{\partial \beta_1}{\partial V_1^{re}} & \frac{\partial \beta_1}{\partial V_1^{im}} & \cdots & \frac{\partial \beta_1}{\partial V_n^{re}} & \frac{\partial \beta_1}{\partial V_n^{re}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial \beta_m}{\partial V_1^{re}} & \frac{\partial \beta_m}{\partial V_1^{im}} & \cdots & \frac{\partial \beta_m}{\partial V_n^{re}} & \frac{\partial \beta_m}{\partial V_n^{re}} \end{bmatrix} \tag{5}$$

where V_i^{re} and V_i^{im} represent the real and imaginary part of complex voltage V_i at bus *i*. The functional relationship between the state variables vector and constrained output vector is established by $\boldsymbol{\beta}$. It is worth mentioning that, aligned with the work in [24], Jacobian $\mathbf{J}_{\boldsymbol{\beta}}$ is dependent on the relationship between voltages and powers. Similarly, the functional relationship between the state vector and the input vector by $\boldsymbol{\alpha}$ can be established. Upon checking the necessary conditions for linearisation, as reported in [24], the linear map between input and output constrained variables can be established. To this end, the knowledge of classical Newton-Raphson load flow study dictates that the Jacobian matrix of input and output vectors can build the linear relationship of the change of state variables (e.g., $\Delta|\mathbf{V}|$) with their respective input and output vectors as given by (6)-(7)

$$\Delta\boldsymbol{\alpha} = \mathbf{J}_{\boldsymbol{\alpha}}(\mathbf{S}).\Delta\mathbf{V} \tag{6}$$

$$\Delta\boldsymbol{\beta} = \mathbf{J}_{\boldsymbol{\beta}}(\mathbf{S}).\Delta\mathbf{V} \tag{7}$$

From (6) and (7) we get,

$$\Delta\boldsymbol{\beta} = \mathbf{J}_{\boldsymbol{\beta}}(\mathbf{s}).\mathbf{J}_{\boldsymbol{\alpha}}^{-1}(\mathbf{S}).\Delta\boldsymbol{\alpha} \tag{8}$$

For a power system containing redundancy which may lead towards singularity, inverse matrix may not exist and hence, the inverse matrix of (8) is transformed into pseudo inverse matrix ($\mathbf{J}_{\boldsymbol{\alpha}}^{\dagger}(\mathbf{S})$), adopting the technique presented in [25]. Now, (8) can be restated as (9)

$$\Delta\boldsymbol{\beta} = \mathbf{J}_{\boldsymbol{\beta}}(\mathbf{S}).\mathbf{J}_{\boldsymbol{\alpha}}^{\dagger}(\mathbf{S}).\Delta\boldsymbol{\alpha} \tag{9}$$

From (9) it is found that the product of two Jacobian matrix establishes the linear map between input and output vectors. The product can be simplified as

$$\mathbf{J}_{\boldsymbol{\alpha\beta}}(\mathbf{S}) = \mathbf{J}_{\boldsymbol{\beta}}(\mathbf{S}).\mathbf{J}_{\boldsymbol{\alpha}}^{\dagger}(\mathbf{S}) \tag{10}$$

Technically, all constrained output variables lie within a specified lower and upper limit, i.e., $\boldsymbol{\beta}_{\min} \le \boldsymbol{\beta}(\boldsymbol{\alpha}) \le \boldsymbol{\beta}_{\max}$; therefore, to express the linear constraints of $\boldsymbol{\beta}(\boldsymbol{\alpha})$, a first-order Taylor Series expansion at an operating point $\mathbf{S_0}$ is exploited, which yields

$$\boldsymbol{\beta}_{min} \le \boldsymbol{\beta}(\boldsymbol{\alpha})|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0} + \mathbf{J}_{\mathbf{w}\boldsymbol{\beta}}(\mathbf{S})\Big|_{\mathbf{S}=\mathbf{S_0}}.\Delta\mathbf{w} \le \boldsymbol{\beta}_{max} \tag{11}$$

In (11), assuming that no change will incur in the network during the time-series computational step-interval, only the change of control variables, $\Delta\mathbf{w}$, is considered and accordingly, $\mathbf{J}_{\boldsymbol{\alpha\beta}}(\mathbf{S})\Big|_{\mathbf{S}=\mathbf{S_0}}$ is replaced by $\mathbf{J}_{\mathbf{w}\boldsymbol{\beta}}(\mathbf{S})\Big|_{\mathbf{S}=\mathbf{S_0}}$. Now, rearranging (11) we get,

$$\begin{bmatrix} \mathbf{J}_{\mathbf{w}\boldsymbol{\beta}}(\mathbf{S})\big|_{\mathbf{S}=\mathbf{S_0}} \\ -\mathbf{J}_{\mathbf{w}\boldsymbol{\beta}}(\mathbf{S})\big|_{\mathbf{S}=\mathbf{S_0}} \end{bmatrix}.\Delta\mathbf{w} \le \begin{bmatrix} \boldsymbol{\beta}_{max} - \boldsymbol{\beta}(\boldsymbol{\alpha})|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0} \\ -\boldsymbol{\beta}_{min} + \boldsymbol{\beta}(\boldsymbol{\alpha})|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0} \end{bmatrix} \tag{12}$$

Note that selection of sensible network conditions and operational variables ensures the convexity of (12). Now, considering voltage as network's capacity constraints, in conjunction with the active and reactive power as control variables, the inequality (12) can be restated as

$$\begin{bmatrix} \mathbf{J}_{(\mathbf{P_0},\mathbf{Q_0})}(\mathbf{S})|_{\mathbf{S}=\mathbf{S_0}} \\ -\mathbf{J}_{(\mathbf{P_0},\mathbf{Q_0})}(\mathbf{S})|_{\mathbf{S}=\mathbf{S_0}} \end{bmatrix}.\begin{bmatrix} \Delta\boldsymbol{P} \\ \Delta\boldsymbol{Q} \end{bmatrix} \le \begin{bmatrix} \boldsymbol{V}_{max} - \boldsymbol{V}(\boldsymbol{\alpha})|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0} \\ -\boldsymbol{V}_{min} + \boldsymbol{V}(\boldsymbol{\alpha})|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0} \end{bmatrix} \tag{13}$$

In addition to (13), constraining the control variables yield the general CCSO problem for the network, which is given by (14)-(16)

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}) \tag{14}$$

$$\text{subject to:} \quad \boldsymbol{A}.\boldsymbol{x} \le \boldsymbol{b} \tag{15}$$

and

$$\boldsymbol{x}_{min} \le \boldsymbol{x} \le \boldsymbol{x}_{max} \tag{16}$$

where $\boldsymbol{A}$ (coefficient matrix) is basically the Jacobian matrix presented in (13), $\boldsymbol{x}$ is the control variables comprising of active and/or reactive power, and $\boldsymbol{b}$ (coefficient vector) contains the limit of constrained output vector (e.g., voltage limit, line current limit, thermal limit, etc.) at an operating point.

2) CCSO for DOE

The general representation of CCSO, as shown in (14)-(16), with consideration of active power to be the only controllable variable, is exploited in this article to deploy the concept of DOE at MV/LV transformers' connection points. However, for DOE allocation, depending on service provider's adopted policy - which could be governed by either technical or soft-equitable or equitable perspectives - the constraints on general optimisation problem are adjusted as detailed below.

(i) *Technical Perspective*: The objective of technical perspective is to maximise the amount of energy transfer by a power network during a given period of time. Therefore, available power allocation strategy under technical perspective tends to follow an optimal allocation algorithm that would allocate capacity to connection-points with the least impact on given operational constraints – maximising how much capacity can be allocated to connection-points as a whole, and completely disregarding the fact that some connection-points are likely to not being allocated any capacity at all. Hence, in consideration of the constraints on control variables, network's physical and operational limits, the objective function of technical perspective maximises the total available export/import power (active/real) in a distribution zone comprising MV-LV lines, transformers, customers/prosumer's DERs and loads. Mathematically, the optimisation problem associated with the technical perspective can be expressed as in (17), (18) and (19) for the allocation of available export power; and (22), (23) and (24) for the import power, see Table I. Note, $\Delta\boldsymbol{P}^{exp}$ and $\Delta\boldsymbol{P}^{imp}$ are vectors containing the available export and import power (control variables). The network's constraints (e.g., voltage, current and transformer capacity limits, etc.) are represented by the inequalities (18) and (23) which exploits the coefficient matrix $\boldsymbol{A}$ and the coefficient vector $\boldsymbol{b}$. All control variables are bounded in a range of their maximum and minimum values as specified by the inequalities (19) and

(24). Please note, in the first-stage of our approach, connection-points refer to distribution transformers' end.

(ii) *Soft-equitable Perspective*: Technical perspective may lead to disparity among power allocation across different connection-points. To reduce the impact, soft-equitable perspective tends to minimise the disparity among allocated power across connection-points by introducing a penalty when, allocated power at a connection-point is deviated from the mean value of aggregate allocated power. Hence, within the constraints of physical and operational limits of the network, the objective of soft-equitable perspective maximises the total available import/export power ensuring a reasonable allocation of active power according to the capacity at each connection point. Thus, the optimisation problem of soft-equitable perspective can be represented as (18), (19) and (20) for the allocation of available export power; (23), (24) and (25) for the allocation of import power, see Table I. The objective function of the soft-equitable perspective, shown in (20), takes account of the capacity of distribution transformers. Hence, the available active power is allocated at connection points considering the ratings of transformers. The coefficients $a_1, \dots, a_n \in \boldsymbol{R}^+$ in (20) represent the nominal capacity/ratings of the connection points.

(iii) *Equitable Perspective*: Like soft-equitable, the objective of equitable perspective is to maximise the available export/import power respecting the constraints on minimum disparity among available export/import power across connection-points, along with the consideration of network's physical and operational limits. However, to deal with the infeasibility case, which might occur under soft-equitable perspective, a virtual parameter has been incorporated in equitable perspective to bound the decision variable within feasible region. Hence, the objective function of equitable perspective maximises the total available import/export power ensuring a compromise between the technical and soft-equitable perspectives. To do so, the virtual parameter, $\lambda_i \in [0, +\infty]$, is introduced as shown in (21) and it can adapt to specific scenarios, allowing flexible allocation of available import/export power at the connection points. Thus, the optimisation problem associated with the equitable perspective can be presented by (17)-(19), (21) for available export power and (22)-(24), (26) for import power.

B. Allocation of DOE for prosumers at LV feeders

Our two-stage approach is a hierarchical and coordinated method. In sub-section II-A, while developing the expressions of DOEs at transformers' end, LV network models are included implicitly through a specialised constraint in the MV level. To this end, the constraint is defined at each distribution transformer and the voltage at their secondary side is extrapolated to the end of a simulated LV network using the operating condition (real and reactive power flow) of each transformer. The simulated network is parameterised to experience a permissible voltage drop, when the corresponding transformer is loaded with 100% of its rated capacity in terms of active power. Thus, incorporating additional voltage band constraints, the expectable voltage rises or drops within the LV networks are implicitly considered. Now, to explicitly allocate the DOEs for each DOE-enabled prosumer in LV networks, an unbalanced AC-OPF approach is exploited. As such, minimising the difference between allowable available export/import power and equitably estimated available export/import power, the optimal allocation of available power, $\Delta p_k^{s,opt}$, for each DOE prosumers can be achieved from (27)-(34).

$$\min_{\Delta p_k^s} \sum_{k\in\mathcal{N}_{h^*}} \sum_{s\in\phi_{k,h^*}} \left(|\Delta p_k^s| - |\Delta p_k^{s,\text{equi}}|\right)^2 \tag{27}$$

subject to:

$$\sum_{k\in\mathcal{N}_{h^*}} \sum_{s\in\phi_{k,h^*}} |\Delta p_k^s| \le |\Delta p_{total}| \tag{28}$$

$$\tilde{p}_k^s + \Delta p_k^s = \Re(v_k^s) \sum_{j\in N} \sum_{\gamma\in\Phi} \left[G_{kj}^{s\gamma} . \Re(v_j^s) - B_{kj}^{s\gamma} . \Im(v_j^s)\right] + \Im(v_k^s) \sum_{j\in N} \sum_{\gamma\in\Phi} \left[G_{kj}^{s\gamma} . \Im(v_j^s) + B_{kj}^{s\gamma} . \Re(v_j^s)\right] \tag{29}$$
$$k \in N_{h^*}, s \in \Phi_{k,h^*}$$

$$\tilde{p}_k^s = \Re(v_k^s) \sum_{j\in N} \sum_{\gamma\in\Phi} \left[G_{kj}^{s\gamma} . \Re(v_j^s) - B_{kj}^{s\gamma} . \Im(v_j^s)\right] + \Im(v_k^s) \sum_{j\in N} \sum_{\gamma\in\Phi} \left[G_{kj}^{s\gamma} . \Im(v_j^s) + B_{kj}^{s\gamma} . \Re(v_j^s)\right] \tag{30}$$
$$k \in N_{h^+}, s \in \Phi_{k,h^+}$$

$$\tilde{p}_k^s = \Re(v_k^s) \sum_{j\in N} \sum_{\gamma\in\Phi} \left[G_{kj}^{s\gamma} . \Re(v_j^s) - B_{kj}^{s\gamma} . \Im(v_j^s)\right] + \Im(v_k^s) \sum_{j\in N} \sum_{\gamma\in\Phi} \left[G_{kj}^{s\gamma} . \Im(v_j^s) + B_{kj}^{s\gamma} . \Re(v_j^s)\right] \tag{31}$$
$$k \in N_{h^-}, s \in \Phi_{k,h^-}$$

$$\tilde{q}_k^s = \Im(v_k^s) \sum_{j\in N} \sum_{\gamma\in\Phi} \left[G_{kj}^{s\gamma} . \Re(v_j^s) - B_{kj}^{s\gamma} . \Im(v_j^s)\right] - \Re(v_k^s) \sum_{j\in N} \sum_{\gamma\in\Phi} \left[G_{kj}^{s\gamma} . \Im(v_j^s) + B_{kj}^{s\gamma} . \Re(v_j^s)\right] \tag{32}$$
$$k \in N \setminus \{0\}, s \in \Phi_k$$

$$\underline{v} \le |v_k^s| \le \bar{v}, \forall k \in \mathcal{N}\setminus\{0\}, s \in \Phi \tag{33}$$

$$|v_0^s| = 1, s \in \Phi \tag{34}$$

where $\Delta p_k^{s,\text{equi}}$ corresponds to the equitable estimation of available export/import power for the house connected at phase $s \in \phi_{k,h^*}$ and bus $k \in \mathrm{N}_{h^*}$. This equitable estimation could be performed by equal/proportional distribution of Δp_{total} among all DOE prosumers.

Please note, in the unbalanced AC OPF formulations (27)-(34), inter-phase coupling is captured in $G_{kj}^{s\gamma}$ and $B_{kj}^{s\gamma}$ terms, where $G_{kj}^{s\gamma}$ and $B_{kj}^{s\gamma}$ correspond to the conductance and susceptance, respectively, between phase s and γ of the line connecting bus k and bus j, where $s, \gamma \in \phi$. Note that, conductance, susceptance incorporates self and mutual effect within and between lines/conductors and hence, inter-phase coupling is explicitly captured via mutual conductance and mutual susceptance parameters that can be calculated from mutual impedance. To do so, the phase impedance matrix is calculated in our approach with the aid of Modified Carson's equations as reported in [26].

The formulations (27)-(34) are based on the nodal current injection method for 3-phase power-flow introduced in [27]. One major assumption in this approach is neglecting the effect of the neutral conductor and focusing on a 3-phase 3-wire network. For 3-phase 4-wire networks, i.e., if the neutral needs to be considered, then, the OPF formulation introduced in [28] can be utilised. Moreover, since the OPF formulations are based on rectangular form of complex numbers, the angle difference between phases is implicitly considered. $G_{kj}^{s\gamma}$ and $B_{kj}^{s\gamma}$ terms obtained from the rectangular form of the complex admittance matrix $Y_{kj}^{s\gamma} \in \mathbb{R}^{3\times3}$ indirectly takes account of angle difference between phases. Also, loads are considered as constant P, Q loads.

TABLE I
Formulation of Optimisation Problems under different Perspectives for allocating available Export and Import power

Allocated available power	Perspectives	Objective Function	Subject to
Export	Technical	$\max_{\Delta P^{exp}} \left(\sum_{i=1}^{n} \Delta P_i^{exp}\right)$ (17)	$\boldsymbol{A}.\Delta\boldsymbol{P}^{exp} \le \boldsymbol{b}$ (18) $0 \le \Delta\boldsymbol{P}^{exp} \le \Delta\boldsymbol{P}_{max}$ (19)
	Soft-equitable	$\max_{\Delta P^{exp}} \left(\sum_{i=1}^{n} \Delta P_i^{exp} - \sum_{i=1}^{n-1}\left(\frac{\Delta P_i^{exp}}{a_i} - \frac{\Delta P_{i+1}^{exp}}{a_{i+1}}\right)^2\right)$ (20) $a_i \ne 0$	
	Equitable	$\max_{\Delta P^{exp}} \left(\sum_{i=1}^{n} \Delta P_i^{exp}\right)$ (17)	$\boldsymbol{A}.\Delta\boldsymbol{P}^{exp} \le \boldsymbol{b}$ (18) $0 \le \Delta\boldsymbol{P}^{exp} \le \Delta\boldsymbol{P}_{max}$ (19) $\frac{\Delta P_i^{exp}}{a_i} + \lambda_i = \frac{\Delta P_{i+1}^{exp}}{a_{i+1}} + \lambda_{i+1};$ (21) $i \in [1, n-1]; a_i \ne 0$
Import	Technical	$\max_{\Delta P^{imp}} \left(-\sum_{i=1}^{n} \Delta P_i^{imp}\right)$ (22)	$\boldsymbol{A}.\Delta\boldsymbol{P}^{imp} \le \boldsymbol{b}$ (23) $\Delta\boldsymbol{P}_{min} \le \Delta\boldsymbol{P}^{imp} \le 0$ (24)
	Soft-equitable	$\max_{\Delta P^{imp}} \left(-\left(\sum_{i=1}^{n} \Delta P_i^{imp} - \sum_{i=1}^{n-1}\left(\frac{\Delta P_i^{imp}}{a_i} - \frac{\Delta P_{i+1}^{imp}}{a_{i+1}}\right)^2\right)\right)$ (25) $a_i \ne 0$	
	Equitable	$\max_{\Delta P^{imp}} \left(-\sum_{i=1}^{n} \Delta P_i^{imp}\right)$ (22)	$\boldsymbol{A}.\Delta\boldsymbol{P}^{imp} \le \boldsymbol{b}$ (23) $\Delta\boldsymbol{P}_{min} \le \Delta\boldsymbol{P}^{imp} \le 0$ (24) $\frac{\Delta P_i^{imp}}{a_i} + \lambda_i = \frac{\Delta P_{i+1}^{imp}}{a_{i+1}} + \lambda_{i+1};$ (26) $i \in [1, n-1]; a_i \ne 0$

III. PROPOSED METHODOLOGY

Fig. 2 outlines the proposed methodology through a series of sequential steps which are to be followed to obtain DOE or available export/import power at a time-interval. The allocated DOE is then published and control signals are sent accordingly to respective DERs' connection points.

The entire approach is explained in two stages. First, the input variables (*P*, *Q*, etc.) of MV network are collected from the available metering infrastructure, and then, the variables are stored in a database in real-time. The network model and its associated files are also saved in a repository. DSSE is employed to extract state variables at all required nodes of the MV network. Please note that state estimation is an established technique which is being used in transmission system to estimate state variables at different nodes/buses with accuracy and precision [20-22]. Upon estimating the state variables, the coefficient matrix $\boldsymbol{A}$ and the coefficient vector $\boldsymbol{b}$ are computed from the expressions (2)-(13). In addition to that, while forming $\boldsymbol{A}$ matrix and $\boldsymbol{b}$ vector, several constraints, such as, voltage band (upper and lower voltage limit) on MV nodes, transformers' neutral and phase currents (nominal), forward and backward maximum active power flow through transformers, ampacity limits of MV feeder, etc., are taken into consideration. The CCSO problem, as expressed in (14)-(16), is solved to estimate DOEs at transformers' end. These DOEs, in other words, the maximum available export/import power at different transformers' zone, could be allocated by adopting different policies, such as, technical, soft-equitable and equitable. Based on the adopted policy, selected expressions from (17)-(26) are solved to yield DOEs at transformers' end.

In the second stage, the allocated available export/import power at each transformer's zone is distributed among the DOE-registered prosumers' DERs. However, prior to dispatch the control signal, a rule-based checking is conducted in software level. To do so, the present operating state of DERs are adjusted with the inclusion of available export/import power. Please note that, first, the total available export/import power are distributed among DOE prosumers evenly or proportional to DERs size/rating. Then, an unbalanced load flow is conducted to check any violation issues, such as, transformers' thermal limit, nodal voltage limit and current limit violations in LV lines, etc. If any violation is assessed, then, an unbalanced AC-OPF is solved by respecting (27)-(34), which basically represent the non-linear non-convex optimisation problem. To obtain a feasible solution of this non-linear programming (NLP) problem, IPOPT v3.12.9 [29] running with linear solver *ma57* has been used in our work. To reduce the optimality error and obtain a suboptimal solution with higher accuracy, the IPOPT options are set as: *acceptable tol:* 10^{-8} and *acceptable constr_viol_tol:* 10^{-12} as suggested in [30]. In short, to solve the OPF problem, the following steps are followed.

- The original LV network is modelled in an open-source software package, namely, OpenDSS [31].
- Relevant data is extracted using Python script.
- The OPF problem is modelled with extracted data and then, initialised in Pyomo [32]. Thereafter, adopting the implementation guidelines from [33], problem is solved with IPOPT [29] solver.

Conversely, the DOE allocation model, described in (27)-(34), can also be incorporated with the convex relaxations of 3-phase OPF formulations as reported in [34-35]. However, careful attention has to be made while applying those convex relaxations for linearisation, in order to avoid the infeasibility in the process of achieving optimal solution that may occur due to convex relaxations.

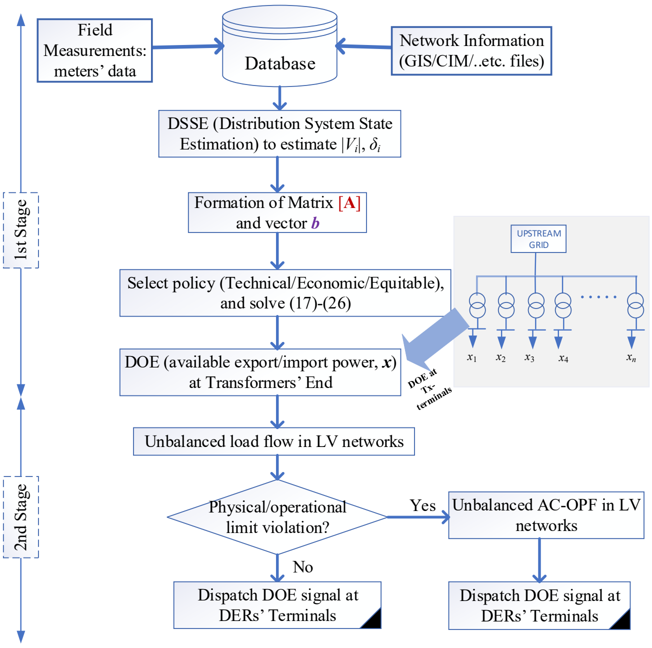

Fig. 2. Proposed two-stage approach to allocate DOE at DERs' connection points.

IV. Case Study

The effectiveness of the proposed methodology is demonstrated in this section by publishing DOE at different connection points, using the data of a real MV-LV integrated distribution network. A real MV feeder supplying a total of 18 distribution transformers of an Australian network are selected for the case study. The network model and its parameters are detailed in Section IV-A. Results and discussions are presented in Section IV-B wherein, DOE allocations at distribution transformers' ends considering technical, soft-equitable and equitable perspectives are first investigated. Then, exploiting the allocated DOE at a transformer's end, time-series operations of a real LV feeder of Australia (containing 102 customers, among which 30% are prosumers), are conducted using OpenDSS and a Python script containing the routines of operations. Further, the results of comparative performance study of proposed DOE-export policy with the existing fixed export policy is elaborated.

A. Integrated MV-LV Network

A real 11kV MV feeder, supplying 18 residential LV networks of Queensland, Australia, is used in our case study. This integrated MV-LV distribution network is owned, maintained and operated by Energex, a distribution utility in Queensland. Fig. 3 shows the single line diagram of this MV-LV network; among the 18 LV networks, one LV network is explored in the diagram, which is used for the performance study demonstrated in Section IV-B(ii).

The head of 11kV MV feeder is supplied by a primary substation through a 33kV/11kV step-down transformer. As the OLTC (On Load Tap Changer) of a primary substation typically keeps the terminal voltage at 1.0 pu, it is reasonable to assume that the voltage at MV feeder's head to be 1.0 pu. Each of the 18 LV networks is fed by an 11kV/0.415kV distribution transformer with the tap-position 1 (off-load). The rated capacities of distribution transformers range from 315-750 kVA, see Fig. 3. The customers of LV networks are distributed along the radial feeder. We assume that the maximum loading of a typical LV network could be represented by a scenario where all the customers in that network would demand 4 kVA at a point in time. Therefore, dividing the rated capacity (in kVA) of a distribution transformer by 4 kVA, the number of customers in an LV network is estimated in this study. An average of 50 customers per LV feeders are connected and average length of the feeder is around 400m (0.4 km). The line parameters of the MV-LV network are shown in Table II.

TABLE II
Parameters of a Real Integrated MV-LV Network

Feeders	No. of feeders	Main path length (km)	R_1 (Ω/km)	X_1 (Ω/km)	R_0 (Ω/km)	X_0 (Ω/km)
MV network: 11 kV	1	6.28	0.142	0.0951	1.227	0.051
LV networks: 415 V	40	0.4 (avg)	0.284	0.255	0.232	0.210

B. Results and Discussions

In the context of DOE, ideally, fairness would be the equal distribution of available power across the distribution transformers' end of MV-LV integrated network. However, the location and size of transformers along the MV feeder is different and hence, consideration of equal available power within network's physical and operational limits would allow very small amount of available power at each connection points, resulting in minimal utilisation of network's available capacity. Therefore, three allocation perspectives (technical, soft-equitable and equitable) have been extensively studied in this sub-section. First, DOE is developed and analysed through a detailed investigation on different allocation perspectives for available export/import power. Then, feasible DOE at LV prosumers' DER-terminals is assessed by a comprehensive performance study. Last, performance of proposed DOE through comparative study followed by scalability and computational efficacy is detailed.

(i) DOE at MV Levels and its allocation perspectives:

Using the proposed methodology, real data from the MV feeder are exploited to obtain DOE at distribution transformers' end. To do so, first, a month-long field measurement data (10-min resolution) are captured from the meters placed at different locations of the MV feeder. Then, DSSE is deployed to estimate state variables, followed by the formation of coefficient Matrix $\boldsymbol{A}$ and coefficient vector $\boldsymbol{b}$. Last, with the available $\boldsymbol{A}$ and $\boldsymbol{b}$, a general CCSO problem is solved taking technical or soft-equitable or equitable perspectives into consideration. The solution yields the DOE at transformers' connection points.

A month-long, September 2020, field-data with 10-min resolution is processed and their corresponding $\boldsymbol{A}$ matrices and $\boldsymbol{b}$ vectors are saved. From (17)-(26), the expressions associated with the technical, soft-equitable and equitable perspectives are exploited taking those $\boldsymbol{A}$ matrixes and $\boldsymbol{b}$ vectors into account. Fig. 4 shows the available import/export limit at the head of MV feeder for 7-days (Sep. 2 - Sep. 9, 2020) under three perspectives. Among 7-days, a representative day (Sep. 9, 2020), with a moderate solar irradiance profile, is selected to explain the results of DOE under different perspectives, see Fig. 5. Figs. 5 (a)-(c) show the available export and import power limit at every 10-min

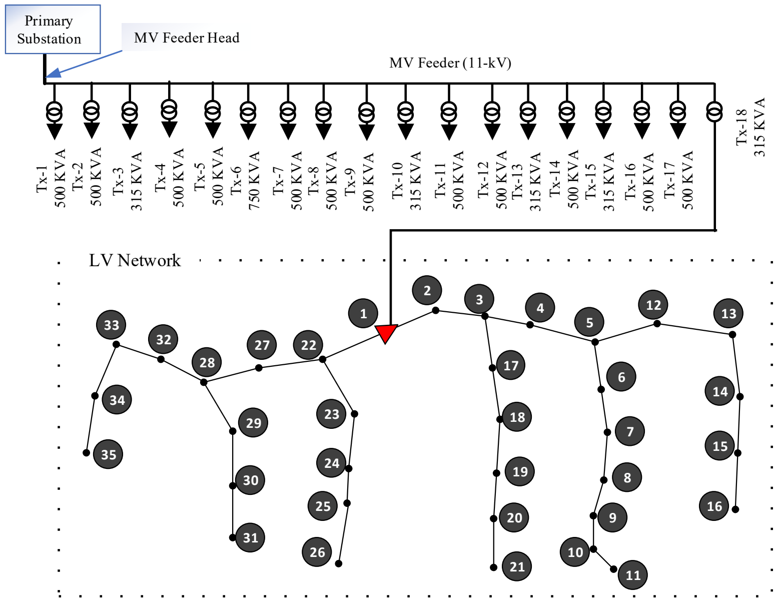

Fig. 3. Diagram of a real integrated MV-LV network located in southeast Queensland, Australia.

interval for each of the 18 distribution transformers end of the network shown in Fig. 3. For example, under soft-equitable perspective, at 12:00 hr, the available export (import) power at Tx-1's (transformer-1's) end is 55 kW (210 kW), Tx-2's end is 52 kW (220 kW), Tx-3's end is 45 kW (150 kW), and so on, see Fig. 5(b).

After self-consumption, PV prosumers' typically export their maximum power to the grid at mid-day or noon due to their pattern of PV generation profile. Therefore, reverse power flow is typically experienced at LV level at noon and it drives the proposed approach to restrict the amount of available export power by eliminating the risk of network's operational and/or physical limit violations. This is evidenced through the available export profile of Figs. 5 (a)-(c), which illustrate that, with the progression of a day from morning (8:00 hr) to mid-day (12:00 hr) or noon (13:00 hr), the available power export-limit or capacity gradually decreases. A similar observation is found for available power-import limit, where gradual restriction on available import-power is imposed during high demand period with minimum/zero PV generation, which typically occurs during late afternoon to evening.

Now, analysing the results of three strategies/perspectives, it is found that technical strategy yields the maximum available export and import power at the head of MV feeder supplying 18 transformers' zone collectively, see Figs. 4-5. However, the relative allocation of available power among 18 transformers' zone are ignored in this strategy, by disregarding the situation where a few or some transformers' zone may be given very small or no power, which may lead to the concern for soft-equitable feasibility, disparity, equity and fairness.

The objective of soft-equitable optimisation is to maximise the overall benefit and/or minimise the negative impact. One example of soft-equitable benefit could be to assign the value of the amount of energy transferred to the power network. Since, the allocated capacity would have the tendency to be exploited in terms of export/import energy relative to the size of transformers capacity. Therefore, in this study, the soft-equitable optimality is achieved by solving the technical optimisation problem considering an additional equality constraint relative to the size of transformers' capacity, see equations (18)-(20), (23)-(25) of Table I. Thus, it is expected that the aggregate available power (import/export) for soft-equitable strategy would be less than technical perspective in most of the time-intervals, which is evident from Figs. 5(a) and 5(b).

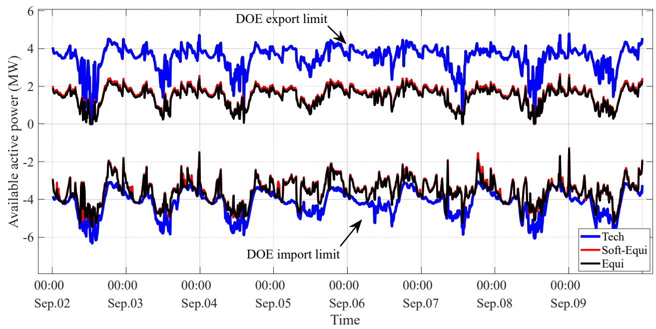

Fig. 4. Available export/import-power at the head of a MV distribution feeder in Queensland, Australia from Sep. 2 - Sep. 9, 2020, considering Technical (Tech), Soft-equitable (Soft-Equi) and Equitable (Equi) perspectives.

The objective of equitable optimisation is to ensure the fairness of allocated capacity across all transformers' zone. As such, the purpose of equitable optimisation, in this study, is to minimise the disparity of allocated capacities across all 18-transformer zone by introducing a virtual parameter λ in the equality constraints, see equations (17)-(19), (21)-(24) and (26) of Table I. Hence, the equitable strategy provides a technically sub-optimal solution which would tend to allocate the lowest overall capacity among three perspectives as evident from Figs. 4-5. Please note that, in the proposed equitable strategy, λ is adjusted to a value of 200, which ensures reasonable fairness by minimising the disparity among available export/import power across transformers.

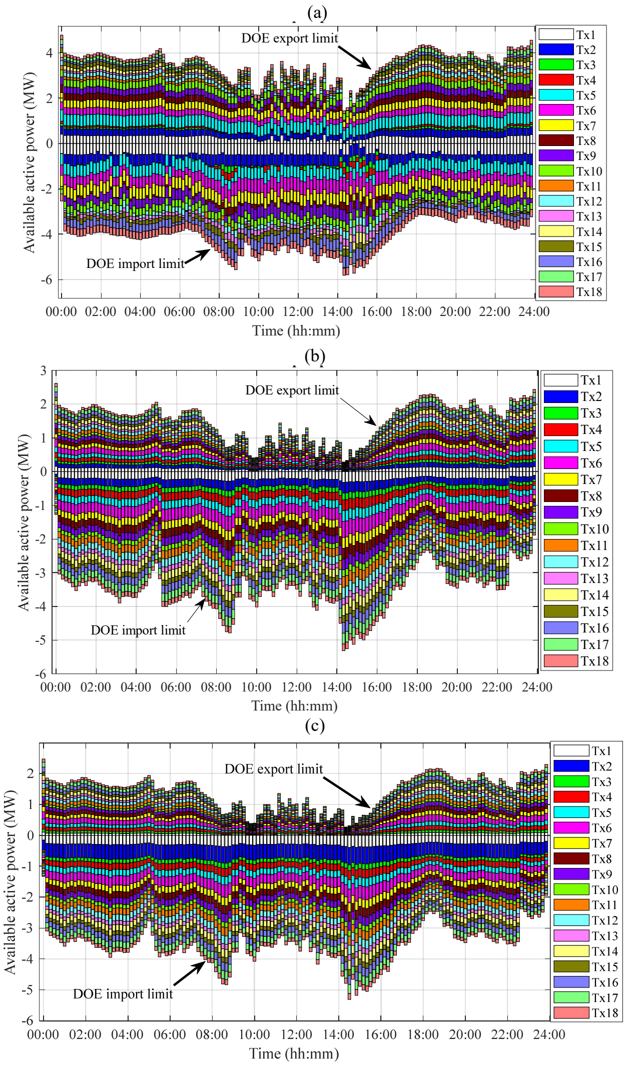

Fig. 5. Available export/import-power (DOE export/import limit) at the connection points of 18 distribution Transformers (Tx1-Tx18) on Sep. 9, 2020, considering 3 perspectives: (a) Technical, (b) Soft-equitable and (c) Equitable.

In addition, to provide an insightful view of relative power allocation, a set of boxplots, specifying the statistical values (maximum, minimum, 75 and 25 percentile) of allocated export power across 18 transformers' zone at every 3-hr, are shown in Fig. 6. The boxplot results infer that, though soft-equitable and equitable solutions allocate less aggregate power in comparison to technical solution, the power allocation across all transformers is relatively closer to one another for soft-equitable and equitable solutions than technical one. Besides, equitable solution guarantees the reasonable fairness by demonstrating the fairest allocation (among the three perspectives) of export power across all transformers.

(ii) Performance study of DOE at LV levels:

This sub-section details the performance study of DOE allocated available export power at DERs' (for PV systems in particular) connection points of LV networks. To this end, first (Case 1), the equitable DOE, obtained from real-field data as presented earlier, is exploited for a time-series operation conducted in a real three-phase LV network (Fig. 3). Then, in Case 2, future operation of DOE - in high renewable-rich network - is simulated through a time-series operation and thus, performance study followed by some discussions are conveyed. Please note, in both case studies, customers of LV networks are categorised into three types - passive customers, fixed-export (FE) prosumers and dynamic-operating-envelope (DOE) prosumers. Passive customers are those who have loads only, FE prosumers have loads and PV-systems operated at fixed export (5-kW) limit, DOE prosumers are those who have loads and PV-systems controlled/regulated by dispatched DOE signal. In this study, 10-kW PV-systems are selected for each DOE prosumers. Further, it is assumed that PV-systems are operated at unity power factor and all loads are inductive with power factor of 0.95. The voltage limit at DERs' connection points comply with the Australian Standard [36], which specifies that the steady-state voltage should remain within +10/-6% of the nominal 230V line-to-neutral value.

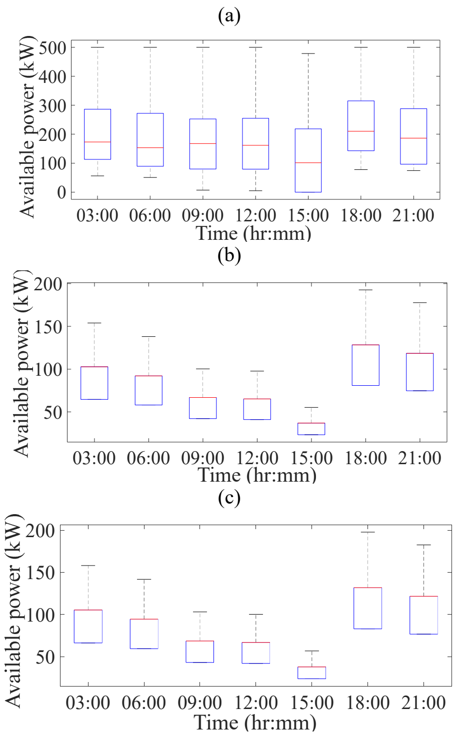

Fig. 6. Statistical illustration of relative allocation of available export-power across 18 transformers within 10-min time interval of every 3-hr on Sep. 9, 2020, under 3 perspectives: (a) Technical, (b) Soft-equitable and (c) Equitable.

Case 1: Considering the equitable DOE at Transformer-18's end, shown in Fig. 3, this case study first investigates the network's performance – evaluated by transformers' capacity utilisation and voltage limit violation issues – under existing 30% FE prosumers and 70% passive customers. A whole-day time-series operation exhibit no voltage limit violations (see Fig. 7(a)) and low transformer's capacity utilisation (see Table III) - indicating under-utilisation of network's resources. Now, assuming 30% of FE prosumers as DOE prosumers, the network's performance is further studied when DOE allocated power-capacity is in place for DOE prosumers. Fig. 7(b) and Table III illustrate no voltage limit violation issues and relatively higher utilisation of transformer's capacity - indicating sensible utilisation of network's resources.

Case 2: Time-series simulation of a high renewable-rich power network is conducted in Case 2 where two scenarios are considered – scenario (*a*): 10% passive customers and 90% FE prosumers and scenario (*b*): 10% passive customers, 60% FE and 30% DOE prosumers. Thus, performance of FE and proposed DOE approach are evaluated through scenario (*a*) and scenario (*b*), respectively. For each scenario, normalised irradiance profile of a typical clear-sky day and cloudy day, shown in Fig. 8, is exploited. For scenario (*a*), voltage limit violations are observed at a number of buses for a significant amount of time in a clear-sky day, see Fig. 9(a); additionally, maximum utilisation of transformer's capacity is reached/exceeded at some periods of time. However, voltage at all buses lie within statutory limit in a cloudy-day for scenario (*a*), see Fig 9(b). For scenario (*b*), no voltage limit violations are found and utilisation of transformer's maximum capacity is not breached, see Figs. 10(a)-(b) and Table III. In reality, the DOE prosumers, in scenario (*b*), help to restrict the voltage within statutory limit. To illustrate this point, the time-series data, containing the DOE allocated export power for a DOE prosumer and the exported power from a FE prosumer, is shown in Fig. 11. As soon as the nodal voltage, line current and transformers' capacity limits are about to breach, DOE does not allow any available power to export and forces the real power export curtailment for DOE prosumer, see Fig. 11(a).

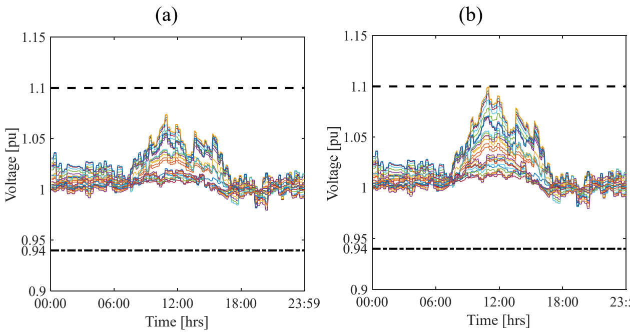

Fig. 7. Voltage profiles at 35-buses of an LV network of the integrated MV-LV system under Case 1, with the presence of (a) 30% fixed-export prosumers and (b) 30% DOE prosumers.

In summary, the results of Case 1 and Case 2 infer that, without violating network's operational and physical limits, DOE ensures the maximum utilisation of network resources in the pathway of achieving high penetration of renewables. However, rapid growth of PV penetration could impose curtailment of power for DOE prosumers to secure network integrity. Therefore, a new business paradigm needs to be explored that can achieve win-win situation for both DOE prosumers and DNOs/aggregators. For example, dynamic tariff of energy could be offered for DOE prosumers when curtailment is imposed, or, the aggregate energy of DOE prosumers could be sold in wholesale energy market, etc.

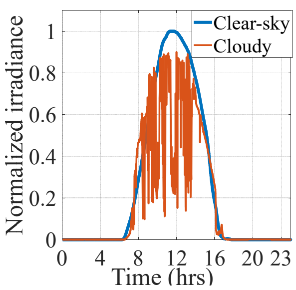

Fig. 8. Normalised solar irradiance profile of a clear-sky and cloudy day.

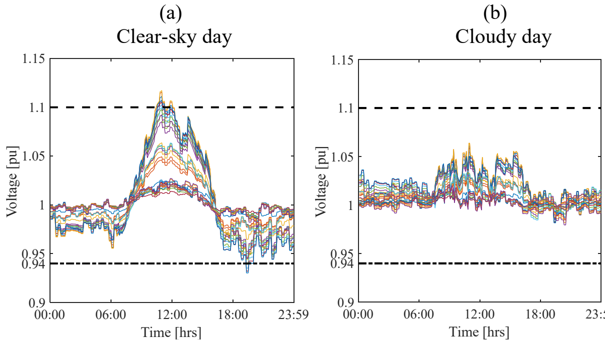

Fig. 9. Voltage profiles at 35-buses of an LV network of the integrated MV-LV system under Case 2, scenario (*a*), during clear-sky and cloudy day.

TABLE III
Performance Study considering Technical Issues in an LV Network

Cases		Fixed Export			Proposed DOE Approach		
		Transformer's utilisation (%)		No. of customers with voltage issues (%)	Transformer's utilisation (%)		No. of customers with voltage issues (%)
		max.	mean		max.	mean	
Case 1		51.5%	18.5%	0%	54.1%	28.4%	0%
Case 2	Clear-sky day	133.3%	66%	21%	108.2%	59.2%	0%
	Cloudy day	51.5%	15.7%	0%	98.5%	49%	0%

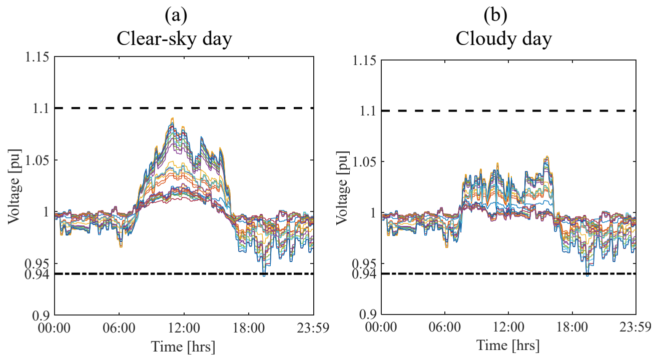

Fig. 10. Voltage profiles at 35-buses of an LV network of the integrated MV-LV system under Case 2, scenario (*b*), during clear-sky and cloudy day.

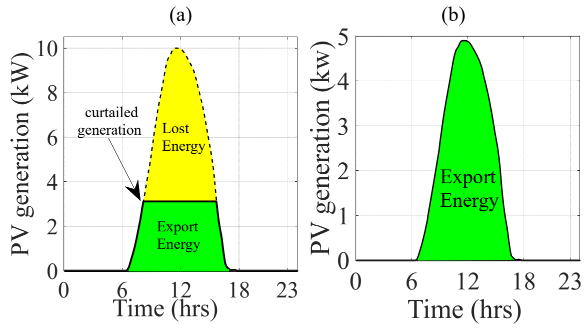

Fig. 11. For Case 2, scenario (*b*), clear-sky day, the PV export-power from (a) a DOE Prosumer and (b) a fixed export prosumer, highlighting the power curtailment of DOE prosumer and the associated loss of PV energy to avoid voltage limit violations in the LV feeders.

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(iii) Comparative Performance study of proposed DOE:

Conducting a comprehensive simulation study on a real MV-LV integrated network of Australia, our proposed DOE approach has demonstrated better performance (in terms of technical issues - transformer utilisation and voltage quality) than fixed-export policy, as illustrated in Table III. Besides, each stage of the proposed two-stage method has an advantage over other reported methods in [11-12], [37].

The reported methods in [11-12] demonstrate accuracy and computational effectiveness in consideration of network integrity and maximum DOE allocations at DERs' terminals. However, the freedom to investigate several perspectives (e.g., technical, soft-equitable and equitable) by network operator is limited in [11-12]. First-stage of the proposed method, on the other hand, offers several options to network operators for DOE allocations, among which any option could be selected considering the real-world implementation aspects along with the compliance of regulatory framework.

The second-stage of our method deals with the unbalanced AC OPF operations for each LV networks of the MV-LV integrated network. These AC OPFs are executed independently and in parallel for each LV network. Thus, the no. of variables in AC OPF are reduced significantly - it can be reduced to 1/20-1/30 times when compared with the consideration of whole MV-LV network covering 20-30 distribution transformers. The reported methods in [11-12], [37] deal with significantly higher number of variables in comparison to our method, as they solve the AC OPF for whole MV-LV network. As reported in [28], computational time for solving unbalanced AC OPF linearly increases with the no. of variables and therefore, our proposed approach offers faster computational platform in comparison to reported methods.

(iv) Performance study of proposed DOE in terms of scalability and computational efficacy:

The studied MV-LV integrated network (see Fig. 3) is a fairly large real network containing 350-bus and 18 MV/LV distribution transformers. Now, in order to demonstrate the scalability of proposed method, validation study is further conducted on a large 900-bus real network from Australia containing 31 distribution transformers (nominal power rating: 200-1,000 kVA), 31 LV networks and approx. 3,500 single-phase customers. Exploiting the real-field data, collected on Aug. 1, 2021, the first-stage of our method publishes the hourly available export/import power under technical perspective (see Fig. 12). Please note, the difference between the DOE processing time for 350-bus and 900-bus network was very small (less than 1 sec). Besides, computational time for second-stage is almost similar for 350-bus and 900-bus network, since unbalanced AC OPF for each LV networks are executed independently and in parallel, resulting in significant reduction of number of variables in AC OPF. Computational time for solving unbalanced AC OPF has a strong relationship with the number of variables and it linearly increases with the no. of variables – it could reach up to 2 sec. for approx. 500-bus network [28]. Numerically, our proposed AC OPF operation - when executed in a computer (Intel corei7, RAM 16 GB, OS – Windows 10) for an LV network shown in Fig. 3, the average computational time was less than 1 sec. Thus, the proposed DOE approach demonstrates the scalability as well as computational efficacy. Besides, the key findings from the results of 900-bus network (shown in Fig. 12) are consistent with the results of 350-bus network (shown in Fig. 5(a)), i.e., the technical perspective shows maximum allocated export/import power as a whole, while disparity of power allocations among different transformer-zones are noticeable.

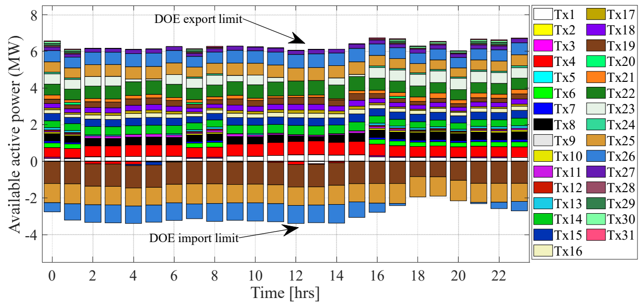

Fig. 12. Exploiting the real-field measurement data from a large network, collected on Aug. 1, 2021, the available export/import-power (DOE export/import limit) at the connection points of 31 distribution Transformers (Tx1-Tx31) considering Technical perspective.

V. Discussions

Dynamic operating envelope (DOE) is a novel technique to utilise existing electricity infrastructure efficiently while appropriately managing network's physical and operational constraints. However, DOE dispatch strategy at prosumers' levels is not a straightforward task and hence, for selecting the suitable strategy, several options should be investigated with careful attention. For example, three such options for DOE dispatch could be - the consideration of: 1) the entire prosumer-premise as a single device, 2) only the inverter-based systems (e.g., PV, energy-storage-systems based DERs, etc.) separately as flexible devices, and 3) a group of inverters inside the LV network's zone. In this article, option 2 has been studied in consideration of PV inverter-based system as flexible devices. However, the proposed platform can be exploited for all 3 options, which would be useful for network operator to select the most suitable DOE dispatch policy in regards to evolving network states and/or business strategies from different stakeholders.

In our approach, while estimating the DOE (available export/import power) at MV level by solving the CCSO problem with different objectives (e.g., technical, equitable), we observed that the export/import power of a connection point had an impact on the export/import capability of other connection points. Therefore, we formulate the CCSO problem considering the feasibility region where the control variables (available export/import power) at all connection points can change to satisfy the objective (equitable or technical) set by the network operator.

In the first-stage of our approach, DOE estimation allows some headroom in consideration of uncertainty of future behaviour of uncontrolled demand and network devices with local autonomous control. However, if network operator requires minimum headroom - which can potentially increase the allocated available power - the setting of autonomous voltage regulating devices need to be incorporated while implementing DOE. For example, one approach could be incorporating the OLTC as a part of optimisation process for DOE calculation.

Uncertainty of renewables is taken care of by allowing some headroom in our approach. Additionally, network operator can increase the frequency of DOE allocations by reducing the time-interval between successive DOE allocations. Also, to eliminate the impact of sudden fast fluctuations from renewables, network operator can restrict the upward ramp-rate of DOE allocation to a reasonable value. That is, for a connection-point, the difference between allocated DOEs (export/import power limit) in successive time-intervals can be restricted to a reasonable value.

VI. Conclusions

This article proposes an approach to generate DOEs for different connection points, e.g., DERs' terminals, transformers' secondary-side, etc., of a distribution network ensuring integrity. The proposed DOE is technology-agnostic and it publishes the available export/import power limit for given connection-points of a network at every 5/10-min interval in a day near real-time. In our proposed method, first, DSSE and CCSO techniques are employed sequentially considering technical, soft-equitable and equitable perspectives for available power allocation at transformers' connection points. Then, the aggregate available power at each transformer-end are distributed among DOE prosumers ensuring reasonable fairness through unbalanced AC-OPF solutions. These AC-OPF programs are executed in parallel for all LV networks of the integrated MV-LV system; thus, reducing the substantial computational time and effort. Using the real-world meter data, the overall execution time of our two-stage approach was less than 1-minute at each time-interval.

The performance of the proposed approach is evaluated using real data from an Australian MV feeder, connected with 18 distribution transformers to supply electricity for LV residential customers. Around 30% of those customers (prosumers) are equipped with solar PV systems. Results show that, technical perspective could allocate maximum aggregate available power (export/import) in most of the time-intervals in a day; however, disparity of allocated power among different transformers' zone are encountered. Equitable perspective, on the other hand, can minimise the disparity of power allocations among transformers' zone at the cost of offering the least aggregate available power to export/import.

Furthermore, in consideration of power export from the DERs' terminals, the performance study of DOE and fixed export policies are carried out by assessing the voltage limit violations and transformers' capacity utilisations in LV networks. Results suggest that renewable-rich distribution grid may experience voltage limit violations at some time-intervals in the middle of a clear-sky day when high percentage of fixed export prosumers exist in the network. However, in such situation, proposed approach restricts the power export for DOE prosumers by forcing power curtailment and thus, helps to keep the voltage and transformers' capacity utilisation within safe operational limit. Therefore, adequate policies, which could be dynamic tariff package, trading in wholesale energy market, etc., need to be undertaken by DNOs/aggregators to incentivise the power curtailment of DOE prosumers that would encourage a new business paradigm to grow.

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VIII. References

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